Instructional Days: 4-6

Topic Description:

Students will apply different strategies to help them make a plan and carry out the plan to solve several problems. These strategies may include (but are not limited to): draw a diagram or picture, make systematic lists, divide and conquer, find the pattern, and guess and check.

**Objectives:**

The students will be able to:

* Name and explain the steps in the problem-solving process.
* Solve a problem by applying the problem-solving process.
* Express a solution using standard design tools.
* Determine if a given solution successfully solves a stated problem.

**Outline of the Lesson:**

* Handshake Activity and Fence Post Activity (20 minutes)
* Explanation of solutions (15 minutes )
* Handshake Activity and reflections (75 minutes)
* Presentations of Handshake Activity (40 minutes)
* Discussion of reflections (15 minutes)

**Student Activities:**

* Work individually on Handshake Activity problem #1 and the Fence Post Activity.
* Volunteers present solutions to problems.
* Work in groups to complete Handshake Activity problem #2.
* Groups give presentations of their problem solutions.
* Discuss reflections on the process.

**Teaching/Learning Strategies:**

* Handshake Activity problem #1 and Fence Post Activity
* Students work individually on Handshake Activity problem #1 and the Fence Post Activity.
* Explanation of solutions
* Have some students volunteer their solutions to the problems.
* Reinforce each step of the problem-solving process by asking questions similar to those from the candy bar problem. You want students to understand that
* Diagrams can be very useful in problems like this to look at a smaller version of the problem before trying to solve for N.
* The fencepost problem is a variation of the candy bar problem or the handshake problem.
* Handshake Activity problem #2 and Reflections
* In groups of 3 or 4, have students discuss, plan, execute, and reflect on Handshake Activity problem #2. Students should follow the directions given in the activity document and write their group’s thoughts on paper.
* Encourage students to make drawings or charts and/or act out the solution. Chart paper can be given to students to display pictures, charts, or graphs. Their job is to explain the process and the solution so that everyone understands.

* Student Presentations
* Each group should be given about 5-10 minutes (depending on the size of the class) to present their plan and solution to the class. Be sure the students show all 4 steps in the problem- solving process.
* Students groups should explain their solutions—why they did what they did
* Discussion of reflections
* Ask students questions that will get them to reflect on why they proceeded in the manner they did. Where did they start? (chart, etc.) What did they do next and why?
* Is their solution complete enough that it could be given to a computer (if they knew the language the computer was using)? Why or why not?

**Resources:**

* Polya, G. How to Solve It. 2nd. Princeton, NJ: Princeton University Press, 2004.
* Handshake and Fencepost Activity
* Handshake Activity #2 Sample Solution

**Handshake and Fencepost Activity**

**For each problem, complete the following information.**

Understanding the problem:

What data or information is known?

What is unknown?

What are the conditions?

Plan the solution: Show your plan for solving this problem.

Carry out the plan: Using your plan, show your work and your solution.

Review and discuss your solution: Reflect on your solution.

**Complete problems #1 and #2 individually.**

1. Handshake Problem #1: Assume there are 20 people in a room, including you. You must shake hands with everyone else in the room. How many hands will you shake? If there are N (where N > 0) people in the room, how many hands will you shake?
2. Fence Post Problem: You need to build one side of a fence that is 12 yards long. This fence will be built with fence posts and rails that connect one fence post to another. If each fence post is 1 yard away from the next fence post, how many fence posts will be needed for this side of the fence? How many fence posts will be needed for a side of a fence that is N (where N > 0) yards long?

**Read and begin planning your solution for problems #3 and #4. These problems will be completed in class tomorrow with your group. Each group will present their solutions to the class.**

1. Handshake Problem #2: Assume there are 10 people in a room, including you. Each person in the room must shake hands one time, and only time, with all the other people in the room. How many handshakes will occur? If there are 20 people in the room, how many handshakes will occur? If there are N (where N > 0) people in the room, how many handshakes will occur?
2. Reflections: Why are problems like these important to learn how to solve? How could this type of solution be of benefit to a carpenter, a chef, a teacher?

**Handshake Activity #2 Sample Solution**

The sample solution is only one possibility. Student groups may have a wide variety of strategies. Ask questions that probe their understanding of the steps of the problem-solving process they used.

**Understanding the problem:**

* What data or information is known? There are 10 people or N people in the room.
* What is unknown? Total number of handshakes
* What are the conditions? Each person must shake hands only one time with all others in the room.   
  All of the handshakes must be added together.

**Plan the solution:** A sample plan could be to describe the plan in words or use a chart or draw a picture and   
then act it out.

Have the people line up in the room. The first person in the line walks down the line and shakes hands with all of the people in the line and then leaves the room. Count the number of handshakes and add to the total (Have students count off).   
The next person in line walks down the line and shakes hands with all of the people left in the line and then leaves the room. Count the number of handshakes and add to the total.   
This continues until there are only 2 people left. They shake hands and leave together. Increase the total by one.

Once the answer is known for 10 people, look for a pattern. Try the process for 5 people, 2 people. See if the pattern holds.

**Carry out the plan:** Using your plan, show your work and your solution.

|  |  |
| --- | --- |
| Person | Shakes Hands with # of people left in line |
| A | 9 |
| B | 8 |
| C | 7 |
| D | 6 |
| E | 5 |
| F | 4 |
| G | 3 |
| H | 2 |
| I | 1 |
| J | 0(last person in line, no one left to shake hands with) |

Now add up the number of handshakes: 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 0 = 45

For 10 people, the answer is the sum of the numbers from 1 to 9, which is 45. 9 is 10 – 1.

For 5 people, the answer is the sum of the numbers from 1 to 4, which is 10. 4 is 5 – 1.

For 2 people, the answer is the sum of the numbers from 1 to 1, which is 1. 1 is 2 – 1.

For N people, the answer is the sum of the numbers from 1 to (N-1).

**Review and discuss your solution:** Each person shakes hands with N – 1 other people. The answer is not N(N-1), though, because each handshake counts as the one handshake for each person, but only one handshake for the total. The Hershey Bar problem helped to start the plan for this problem, but I needed to adjust the plan to only allow one handshake between each pair in the room.

So the 10 people make 9 handshakes each, but each handshake happens between 2 people, and can only be counted once. I could "divide" the handshake and let each person count the handshake as a 1/2 handshake. So 10 people make 9 half-handshakes each = 45 handshakes.

N people make N-1 half-handshakes each = N(N-1)/2

The sum of the numbers from 1 to N-1 = N(N-1)/2